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High-Order Methods for MHD

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Summary

Magnetohydrodynamics (MHD) is central to many challenging physics problems of importance to the DOE Office of Science. We have developed a state-of-the-art code that features high-order numerical discretizations and multigrid solvers capable of scaling to thousands of processors to simulate MHD in complex domains.

MHD governs the motion and stability of many physical phenomena of interest to DOE, including astrophysical plasmas, geo- and solar dynamos, fusion plasmas, liquid metal cooling systems in nuclear reactors, and liquid metal plasma-facing material in tokamak side-walls and diverters. Most of these applications are dominated by convective transport and operate at high hydrodynamic and magnetic Reynolds numbers, Re and Rm, respectively. The governing physics is thus highly nonlinear and is essentially nondissipative. In addition, many of these applications involve complex domains that preclude the use of traditional global spectral methods.

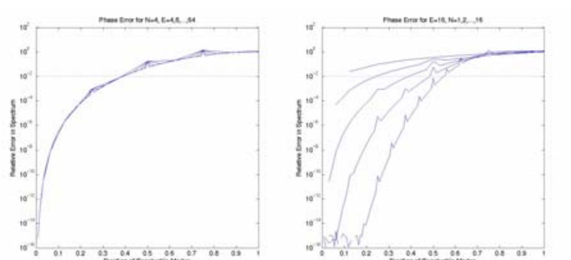

Figure 1. Phase error vs. fraction of resolvable modes for h-refinement (left) and p-refinement (right) for $u_t + u_x = 0$. The fraction of resolvable modes is increased only with increased order (p-refinement), which also yields rapid error reduction.

Working with University of Chicago astrophysicists Fausto Cattaneo and Aleks Obabko, Argonne researchers have developed a numerical code for simulation of liquid metal MHD that couples state-of-the-art high-order numerical methods with the geometric flexibility required for the challenging MHD problems facing the DOE and scientific community.

Our MHD code is based on Argonne's hydrodynamics code Nek5000, which is a past Gordon Bell Prize winner that readily scales to thousands of processors. Nek5000 is based on the spectral element method (SEM), a high-order weighted residual technique that combines the geometric flexibility of finite elements with the rapid convegence and tensor-product efficiencies of global spectral methods. As with the finite element method, functions in the SEM are represented on compactly supported subdomains (elements), thereby simplifying implementation of complex boundary conditions. Grid refinement in the SEM is achieved by increasing the order of the polynomial representation within each element, with typical orders in the range of N= 8–16. The use of such high order minimizes numerical dissipation and dispersion and is important for high-

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Reynolds number applications where high wave-number error components are only weakly damped by physical viscosity. This point is illustrated in Fig. 1, which shows that increasing approximation order leads to efficient use of computational resources. For modest error tolerances, a fivefold reduction in the number of gridpoints *per space dimension* is achievable by going from linear elements to 12^{th}-order elements.

In addition to minimizing MHD discretization errors, we have made significant strides in stabilized methods, on multigrid solvers, and porting to terascale platforms. Of particular importance to MHD is the development of dealiased quadrature rules that ensure energy conservation (Fig. 2, left). We have recently developed spectral element multigrid techniques that have proven to be two to three times faster than our earlier multilevel Schwarz methods across a range of applications. We have ported the new MHD code to NERSC's Seaborg platform (for which we received a two million node-hour allocation as part of a DOE 2005 INCITE Award) and to Argonne's 2048-processor IBM BG/L. Nek5000 is currently sustaining > .9 Tflops on BG/L. We expect this performance to improve significantly with recently developed matrix-matrix product kernels that make better utilization of the BG/L dual-core processors.

We are working with experimentalists led by Hantao Ji at Princeton Plasma Physics Laboratory (PPPL) to study the magneto-rotational instabilities that are believed to be responsible for the generation of turbulence in (magnetized) accretion disks. (The gravitational energy released by accretion—the in-fall of material into a central potential well—is believed to power many of the energetic phenomena observed in the universe.) Rotational flows that are nominally stable can become destabilized through the introduction of a weak magnetic field. Our initial simulations have demonstrated that this is indeed the case for a particular choice of Re and Rm. Because the ratio Rm/Re is a fixed property of the fluid medium, experiments are limited to relatively small values of Rm and thus can not reach the highly nonlinear magnetic regime. The computations are restricted to smaller values of Re, but are able to simulate at much larger values of Rm. The numerics and experiments are thus complementary and will be able to map out a larger region of the Re – Rm parameter space than would be possible using a single mode of investigation.

We have completed a sequence of axisymmetric hydrodynamic and MHD simulations and have initiated a sequence of hydrodynamic simulations (Fig. 2, right) that will make allow comparison with the PPPL data. The PPPL collaboration has led to a new DOE funded project on the effects of MHD on the stability of free-surface flows.

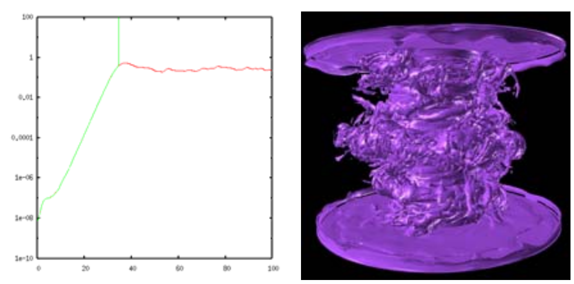

Figure 2: (left) History of magnetic energy in an MHD benchmark for aliased (green) and dealiased (red) nonlinear evaluations. In the saturated nonlinear state, the aliased case is numerically unstable. (right) Simulation of a turbulent flow-field for the PPPL experimental configuration.

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